A study of thermal and mechanical behaviour for the optimal design of automotive disc brakes

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Abstract: The thermal distortion induced by thermoelastic instability (TEI) results in hot spots on the surface of the brake disc. This can incur low-frequency vibration known as judder. In pad-induced squeal noise, mode coupling occurs owing to the variation in the friction coefficient between the disc and pad, inducing high-frequency noise. Through a coupled analysis of hot spots and squeal phenomena, an optimum disc and pad design can be designed for higher thermal and mechanical performance. In this study, numerical and experimental analyses are performed in accordance with disc thickness, pressurization type of caliper, and lining arc length, considering thermal and mechanical instability simultaneously. Thermal deformation and pressure distribution are calculated using a finite element analysis (FEA). For evaluating TEI performance, experiments are performed using a chassis dynamometer and a high-speed infrared camera, and the results are correlated with FEA results. A complex eigenvalue analysis is conducted to evaluate mechanical instability using an FEA. Modal testing and simulations are conducted to correlate a real model and an FE model, and the corrected simulation results are applied for a complex eigenvalue problem to analyse coupled modes according to rotor and pad shapes. The results on disc brake performances considering the disc and pad design are discussed in terms of hot spots and squeal problems.

Keywords: thermoelastic instability, hot spots, judder vibration, pad-induced squeal noise, thermal instability, mechanical instability, complex eigenvalue analysis

1 INTRODUCTION

Disc brake systems are prone to noise and vibration problems arising because of the severe thermal and mechanical loads applied to stop the vehicle. This noise, vibration, and harshness (NVH) phenomenon is not only uncomfortable but also dangerous. In addition, thermal variations, which occur by frictional heat, generate mechanical pressure variation between the disc and lining owing to a change in the friction coefficient between the rotor and pad lining materials. The pressure variation is a non-linear phenomenon, as friction phenomena are generally non-linear coupled problems. In particular, hot judder vibration and squeal noise are non-linear coupled phenomena in automotive disc brake systems. These phenomena also share core design factors such as the pressure distribution between rotor and lining, the shape of the rotor and pad (stiffness), the number of air vents, cooling performance, and friction variation. Hence, judder and squeal should be considered and analysed at the same time for optimization of disc brake design.

When severe friction heating occurs over a certain sliding speed (critical speed) between the rotor and pad, thermoelastic distortion occurs [1, 2]. Some critical factors such as the critical speed, external temperature, run-out, and disc thickness variation (DTV) can cause thermal distortion of the brake disc. In addition, frequent braking also induces high thermal deformation in the brake disc. These conditions cause relatively high thermal distortion and hot spots, which are one of the origins of hot judder vibration [3]. Hot judder vibration, a relatively high-magnitude but low-frequency vibration (10–30 Hz), is transmitted from the disc brake system to

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the driver through the hub, suspension, steering wheel, brake pedal, and floor [4]. Moreover, combined with frequent and high-temperature hot spots, it can easily lead to material damage, including the formation of thermal cracks on the disc surface. The hot-spots phenomenon, also known as frictionally excited thermoelastic instability (TEI), was first observed and explained by Barber [3] for sliding systems involving frictional heating. Lee and Barber [5] solved the TEI problem by assuming that perturbation in the temperature and stress field increases exponentially with time. They showed that the onset of instability is always characterized by an antisymmetric perturbation corresponding to a circumferentially buckled deformation mode that leads to hot spots at alternating locations on the two sides of the disc. Furthermore, using the automotive disc model, they showed that critical speeds calculated by two half-space models overestimate the experiment. Yeo and Barber [6] derived a finite element (FE) formulation of the perturbation method in which the linearity of the governing equations is exploited to obtain separated-variable solutions for the perturbation with exponential variation in time. Yi et al. [7] solved the TEI problem for the geometry of disc brakes and clutches by using a finite element analysis (FEA). They explained that the dominant wavelength (hot-spot spacing) and critical speed are not substantially affected by the three-dimensional (3D) effects, being well predicted by a two-dimensional (2D) analysis, excluding the bending mode. Yi et al. [8] verified the TEI problem of a brake disc through an FEA and experiments. On the basis of TEI theory, a standard design criterion for brake systems subjected to hot judder can be constructed.

Despite efforts to reduce or eliminate its occurrence, squeal noise presents another major brake NVH problem throughout the automotive industry. It is a high-frequency noise produced when the driver decelerates and/or stops the vehicle at a low speed. Because it is coupled to the brake system, squeal cannot be solved easily. Therefore, the squeal problem should be evaluated carefully. There are two main approaches to simulate and analyse disc brake squeal using FEA methods: one is non-linear transient simulation or dynamic transient analysis, and the other is linear or non-linear stability analysis using complex mode evaluation [9–13]. Both methods have their advantages and limitations; to analyse and predict squeal accurately, both need to have good correlation between simulation and test in terms of modal characteristics of rotor and other brake components [9]. There have been many research studies through simulation and experimental approaches. Dessouki et al. [14] characterized squeal into caliper-bracket-induced (2–6.5 kHz), pad-induced (4–11 kHz), and rotor-induced (7–16 kHz) classes. Using FEA, Junior et al. [15] studied the effects of some operating parameters such as friction coefficients, material properties, wear, and insulators on the stability characteristics of a disc brake system. Fieldhouse [16] studied some specific noise frequencies in accordance with the shapes of the pad, and explained that dynamic instability can be predicted and developed by a method where the caliper operates as a one-pot or two-pot type. Kung et al. [17] studied a low-frequency squeal problem using a complex eigenvalue method, reporting that this approach is effective for squeal analysis. Dihau and Jiang [18] studied mode couplings to solve a complex eigenvalue problem using the FE method. Triches et al. [19] used modal analysis techniques to select appropriate brake dampers to reduce brake squeal. Kung et al. [20] analysed the squeal problem on a front disc brake using the new complex eigenvalue capability in ABAQUS/Standard. They showed that contact conditions and other non-linear effects from the preloading are taken into account to formulate the base state for the complex eigensolution. Giannini and Sestieri [21] studied the stability of the model using complex eigenvalue analysis and experiments. They discussed the key role of the disc and the pad dynamics.

In this study, TEI and mechanical instability are investigated in accordance with three rotor specimens: lining arc lengths of one-pot pressurization and two-caliper pressurization types (one-pot and two-pot types). A brake dynamometer and a high-speed infrared camera are used for the TEI analysis. The coning angle is formed in accordance with brake disc shapes such as hat and neck, and this angle alters the contact pressure distribution between the disc and the pad [22–29], and is one of the main factors that change the pressure distribution of the lining. The pressure distribution strongly affects thermal and mechanical instability as well as thermal performance. First, thermal deformations and coning angles under the constant temperature change are calculated by consideration of the geometry of rotors using an FE commercial code, ABAQUS 6.6. The pressure distributions of the pad in accordance with caliper pressurization types were calculated, and the results were analysed and compared with the TEI experimental results. The
analytical results for the critical speed are obtained using TEI-based commercial software, HOTSPOT-TER™. The results obtained from experimental and analytical methods are compared and analysed. To conduct a complex eigenvalue analysis, natural frequencies and modes are determined by modal tests and FEA of discs and pads. Using FEA, the complex eigenvalue problem is solved in accordance with the disc thicknesses, lining arc lengths, and pressurization type to determine unstable coupled modes and difference of mechanical instability for estimating mechanical performance. Finally, the thermal and mechanical behaviours for the optimal design are evaluated and analysed using these results. Figure 1 shows the optimal design factors for the disc brake system. The design factors in the dotted box were considered for optimal disc brake design in this study.

2 THERMAL DEFORMATION AND STRUCTURAL FEA

The disc brake rotor consists of a hat section, neck section, air vent, and outboard and inboard plates (Fig. 2(a)). Frictional heat, which occurs because of contact between the inboard and outboard plates and the brake pad, is assumed to be the main heat source of the rotor. Thermal processes such as heat conduction, convection, and radiation generate temperature gradients on the rotor and cause thermal deformation. In addition, the inboard and outboard thicknesses govern not only the local heat concentration on the contact surface due to conduction velocity difference of the thickness direction, but also the rotor stiffness (especially out of plane). Figures 2(b) and (c) show the geometry of the rotor in accordance with the rotor thicknesses and caliper pressurization types used in this study. The three employed geometric models are a base model, 2t–0t model, and 2t–2t model. All three rotors have a diameter of 254 mm. In the 2t–0t rotor model, the outboard plate thickness is reduced by 2 mm. Similarly, in the 2t–2t rotor model, both the inboard and the outboard blade thicknesses are reduced by 2 mm.

2.1 FEA model and boundary conditions

The thermal deformation of the rotor due to frictional heating produces a non-uniform pressure distribution between the disc and pad. The contact pressure between the rotor and pad highly affects the TEI and the mechanical instability. The frictional heat source is a function of the normal force, friction coefficient, and relative velocity. This eccentric pressure distribution between the disc and pad arises because of the coning angle, run-out, DTV, and pressurization type of caliper and can lead to rapid non-uniform heat generation. Among these factors, the coning angle can be reduced by appropriate rotor shape design. For this, thermal deformation and pressure distribution FEA are performed. Figures 3(a), (b), and (c) show the coning angle and a 3D FE model for a
thermal deformation and structural analysis of the disc brake. The dark grey surfaces in Fig. 3(b) show the location of applied pressures on the back plate.

The coning angle has no fixed value because the temperature distribution of the rotor varies during driving. Generally, experiment and FEA through regulated braking condition are performed to achieve coning angles, thermal stress, and thermal capacity \([25, 28, 29]\). However, in this study, a uniform temperature distribution of 100°C is assumed in the thermal FEA for finding the relative coning angle according to the rotor thicknesses. Through the simulation results, thermal behaviours were evaluated and analysed in accordance with the inboard and outboard thickness differences. Non-linear pressure distribution FEA was performed for analysing the TEI and mechanical instability in accordance with the pressurization condition and pad shape. In order to produce a pressure distribution on the pad corresponding to one-pot and two-pot pressurization calipers, pressure conditions reflecting the caliper piston shape are applied. Both static and dynamic conditions are applied. In the static condition, the rotation speed is zero and a static pressure of 1.5 MPa is applied. In the dynamic condition, the rotation speed of the rotor, the pressure magnitude, and the friction coefficient are 10 r/min, 1.5 MPa, and 0.4 respectively. In the TEI analysis, the pressure distribution of the pad can be used to predict the effective pressure and the effective lining arc length, which governs the number of hot spots and critical speed \([30]\). The number of hot spots and critical speed are closely related to the lining arc length \([5]\). Therefore, utilizing the concept of effective pressure and effective lining arc length can make the TEI simulation more accurate. A thermal deformation and structural analysis is performed to calculate the coning angle and pressure distribution using ABAQUS 6.6, a commercial FEA package.

2.2 Simulation results

2.2.1 Thermal deformation analysis

The coning angles in accordance with the thicknesses of the brake disc were calculated by a thermal...
structural FEA. Table 1 shows the simulation results for the coning angle and deflection of the inboard plate at 100°C. The relative ratio of coning angle is the ratio between the coning angle of the rotor specimen and the base rotor. The results show that the coning angle has a relatively low absolute value. However, the relative coning angle difference is more important than the absolute coning angle because of the constant-temperature boundary condition needed to achieve a relative thermal deformation difference according to the rotor shape. Under the standard of the base rotor, the 2t–0t model shows the smallest ratio of coning angle. This result indicates that the geometric difference of the neck section strongly acts on the geometric constraint, which makes the deformation difference. Therefore, the inboard and outboard thickness difference markedly affects not only the structural characteristic but also the thermal deformation behaviour. In a real automobile, under severe thermal and mechanical loads, the deflections of the rotor edges have relatively high values compared with the non-uniform pressure effect which occurred by the DTV and run-out. Therefore, it should not be neglected. Although the run-out and DTV also increase the local pressure between the disc and pad, they do not depend on the rotor cross-section shape. Therefore, for optimal design of the disc, it is necessary to consider the thermal deformation and coning angle of the rotor as well as the thermal capacity.

2.2.2 Structural analysis

A uniform pressure distribution between the rotor and lining is one of the most important factors in the optimal design. A uniform pressure distribution implies larger contact area, broader contact heat generation, and stiffer lining, which can result in hot spots and more noise in the brake system. A uniform pressure distribution can also cause brake NVH problems. However, to obtain highly efficient braking force under stability, a uniform pressure distribution is more stable than a relatively non-uniform pressure distribution. In addition, it is associated with relatively better thermal and mechanical behaviours and wear performance due to the lower pressure magnitude in comparison with a non-uniform pressure distribution. The variables of caliper pressurization type and lining arc length can alter the pressure distribution, contact area, and lining stiffness at braking. Therefore, the pressure distribution between the disc and pad should be analysed according to the pressurization type and pad length for optimal design, especially in relation to the hot-spot phenomenon. Figure 4 shows the pressure distribution of the outboard pad at 100 per cent, 90 per cent, and 80 per cent lining arc length in a one-pot type caliper. Simulation conditions are as follows: the rotation speed of the rotor and pressure magnitude are zero and 1.5 MPa respectively at the stop condition; the rotation speed of the rotor, pressure magnitude, and friction coefficient are 10 r/min, 1.5 MPa, and 0.4 at the rotation condition respectively. Table 2 shows the simulation conditions for the pressure distribution and complex eigenvalue analysis. The graphs in Fig. 4 show the stress distribution according to the path on the centre of the lining, denoted by a dotted curve. The vertical axis is the pressure ratio $P/P_{\text{max}}$ and the

![Fig. 3](image1.png)

**Table 1** Simulation results for coning angles and deflections of the rotor edge

<table>
<thead>
<tr>
<th>Disc specimens</th>
<th>Coning angle (deg)</th>
<th>Deflection (μm)</th>
<th>Relative ratio of coning angle (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>3.88 x 10^{-1}</td>
<td>122</td>
<td>100.0</td>
</tr>
<tr>
<td>2t–0t</td>
<td>1.79 x 10^{-1}</td>
<td>40</td>
<td>32.8</td>
</tr>
<tr>
<td>2t–2t</td>
<td>2.39 x 10^{-1}</td>
<td>65</td>
<td>53.3</td>
</tr>
</tbody>
</table>
The horizontal axis is the lining arc length ratio $L/L_{\text{max}}$. The black curve, dashed curve, and dotted line are the pad pressure ratio at the stop status, the pad pressure ratio at the rotation status of the rotor ($\omega = 10 \text{ r/min}$), and the effective pad pressure ratio, the integration value of which corresponds to the black curve. The results indicate that, as the lining arc length is reduced, the effective pad pressure ratio is increased. For the one-pot pressurization type, the dead zone of the lining results in a shorter effective lining arc length. The dead zone indicates a relatively low-pressure distribution location. This result is highly related to the TEI analysis, because the critical speed is fairly dependent on the lining arc length. At the rotation condition, the stress distribution is moved along the rotation direction. In addition, the pressure distribution difference between circumferential inside and outside occurred owing to the relative velocity difference of circumferential direction.

**Figure 5** shows the pressure distribution of the lining in the two-pot-type caliper. The effective pad pressure and effective lining arc length are higher than in the case of the one-pot type at the centre path. Regarding the TEI, higher effective lining arc length or effective pressure implies lower critical speed [30]. The pressure distribution is relatively uniform compared with the case of the one-pot type. However, the stress distribution of the upper path is comparatively non-uniform, which can cause a dead zone. The upper path has a higher relative velocity than the lower path and this velocity difference can cause a pressure difference on the contact area of the rotor. In the rotation condition, the pressure distribution had the same tendency as the one-pot-type results. In particular, the pressure was concentrated in the radial direction and the pressure distribution of inside and outside circumference occurred in the circumferential direction owing to

**Figure 4** Pressure distribution of the outboard pad: (a) compressive stress at 100 per cent lining arc length, (b) compressive stress at 90 per cent lining arc length, and (c) compressive stress at 80 per cent lining arc length, all in a one-pot-type caliper.

**Table 2** Simulation conditions for pressure distribution and complex eigenvalue analysis

<table>
<thead>
<tr>
<th>Rotor specimens</th>
<th>Base, 2t–0t, 2t–2t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pad specimens</td>
<td>100%, 90%, and 80% lining arc length in a one-pot caliper</td>
</tr>
<tr>
<td>Caliper types</td>
<td>one-pot and two-pot</td>
</tr>
<tr>
<td>Rotation speed</td>
<td>0, 10 r/min</td>
</tr>
<tr>
<td>Applied pressure</td>
<td>1.5 MPa</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td>0.4</td>
</tr>
</tbody>
</table>
the relative velocity. One of the solutions for reducing non-uniform heat inserts due to the relative velocity difference between the circumferential direction inside and outside on the two-pot-type caliper is to reduce the size of the pressurization piston in the radial direction. The lining stiffness can be increased in comparison with the one-pot type when braking is applied under the same pressurization force. This results in coupled unstable modes, which lead to squeal phenomena. Uniform contact pressure distribution also affects braking stability. Therefore, not only the relative velocity and pressure difference in the radial and circumferential direction but also the pressurization type of caliper should be simultaneously considered for optimal design of the pad shape.

3 THERMOELASTIC INSTABILITY

3.1 Experiments

The critical speed is a fundamental criterion for evaluating TEI performance. Speeds higher than the critical speed can lead to hot spots. Hot spots on the disc surface can cause not only hot judder vibration but also wear, material damage, and thermal cracking. Therefore, evaluation of TEI performance is essential for optimization. The critical speed is determined in accordance with rotor thicknesses, pressurization types, and master cylinder pressures respectively. Three kinds of disc having different thicknesses and two kinds of pad were used in the experiments. A brake dynamometer is used to apply braking. The master cylinder pressures were 1.5 and 2.0 bar. For observing hot spots, a high-speed infrared camera was used. Base, 2t–0t, and 2t–2t specimens were assessed. A one-pot pressurization-type pad of full lining arc length, i.e. 46.4°, was used. Two-pot pressurization-type pad specimens of full (49.8°) and 80 per cent (39.2°) lining arc lengths were also used. Table 3 shows the experimental conditions of critical speed according to the pad specimens and the pressure of the master cylinder.

The experimental procedure for finding the critical speed is as follows.

Step 1. Set up the initial rotation speed of chassis dynamometer (in this experiment, \( \omega_{\text{initial}} \) is 800 r/min).
Step 2. Apply the pressure and measure the temperature distribution of the rotor surface.

Step 3. Increase or decrease the minimum speed by 100 r/min and repeat step 2 until hot-spot occurrence.

Step 4. Decrease the rotation speed by 100 r/min, which leads to the occurrence of a hot spot, and then increase the rotation speed by 10 r/min.

Step 5. Apply the pressure and measure the temperature distribution of the rotor surface.

Step 6. Increase the rotation speed by 10 r/min and repeat step 5 until hot-spot occurrence.

Step 7. The critical speed can be achieved.

Eighteen experiments with different rotor thicknesses and pad specimens, and 12 experiments with different pad specimens, were carried out, i.e. three experiments per pressure setting. The pressures were applied manually. For all experiments, the initial temperature was 15°C and the run-out was set to be less than 10 μm. The experimental equipment is shown in Fig. 6.

### Table 3

<table>
<thead>
<tr>
<th>Rotor specimens</th>
<th>Pad arc lengths</th>
<th>Pressures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base, 2t–0t, 2t–2t</td>
<td>46.4° (100% lining arc length; one-pot caliper type)</td>
<td>1.5 bar, 2.0 bar</td>
</tr>
<tr>
<td></td>
<td>49.8° (100% lining arc length; two-pot caliper type)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>39.2° (80% lining arc length; two-pot caliper type)</td>
<td></td>
</tr>
</tbody>
</table>

### 3.2 Finite element analysis for predicting TEI

#### 3.2.1 Theoretical formulation

Braking with an automotive disc brake can be expressed by heat conduction and thermoelastic coupling problems. The frictional axisymmetric 3D heat transfer equation of two layers (Fig. 7) can be reduced to a 2D equation in the braking system. For a disc brake system, it is assumed that solution of the temperature field grows exponentially over time, according to

\[ T(x, y, t) = T_0(x, y) + \Re(e^{bt + jmy}) \phi(x) \]  

where \( b \) is the complex exponential growth rate, \( m \) is the wave number, and \( \Re \) represents the real part of a complex value. The wave number is defined as the number of oscillations for perturbation within a length of \( 2\pi m \) and \( m \) can be any positive real value. \( T_0(x, y) \) is the steady state temperature solution and always satisfies the heat equation. To solve the thermoelastic contact problem, the pressure distribution \( p(x, y, t) \) must be determined as a function of instantaneous temperature field \( T(x, y, t) \). Yi et al. [31] explained various methods of determining this relation. Discretizing the heat equation in a matrix form using standard Galerkin FE formulation and applying the thermoelasticity relation yields

\[ (K + bH) \Theta = 0 \]  

which is a generalized linear eigenvalue equation for the exponential growth rate \( b \). When TEI is indicated, at least one eigenvalue has a positive real part.

![Fig. 6](a) Experimental equipment: chassis-dynamometer and high-speed infrared camera. The infrared camera is placed perpendicular to the disc surface.
Therefore, the critical speed can be obtained using iteration and a feedback method until a positive real value of the eigensolution is achieved [8].

### 3.2.2 FEA model and conditions

Critical speeds in accordance with brake disc thicknesses and pad specimens were calculated using HOTSPOTTER, a TEI analysis program developed by Yi et al. [7]. The FE model consists of a back plate (steel), lining (non-asbestos organic (NAO) material), and a rotor (cast iron). The lining arc lengths of one-pot and two-pot type were 46.4° and 49.8° respectively. The thicknesses of the lining and back-plate were 8 mm and 6 mm, and the thicknesses of the inboard and outboard plates were 6 mm for the base rotor specimen. The friction coefficient was 0.4. In the TEI analysis, the critical speeds varied with the number of hot spots. The number of hot spots determined from the experiments and the axisymmetric condition were used in this simulation. Table 4 shows the material properties used in this study. The experimental results were given by the manufacturer while the thermal properties were from the work of Bijwe and Kumar [32]. The other material properties were taken from the experimental results obtained by Yi et al. [8].

### 3.3 Results and discussions

To investigate the disc brake performance in terms of the TEI, the critical speeds were measured with various disc thicknesses, pad specimens, and pressures of the master cylinder respectively. The total number of experiments was 30, which were divided into four groups. Figure 8 shows the temperature distributions for various rotor thicknesses, pressures of the master cylinder, and lining arc lengths, acquired by a high-speed infrared camera. Six hot spots were observed in all cases except for the case of 2.0 bar master cylinder pressure and 80 per cent lining arc length in a two-pot pressurization-type caliper.

Figure 9 shows the graphs of the simulation and experimental results on the critical speed for various thicknesses of the brake disc, pad specimens, and pressures of the master cylinder respectively. The results reveal that a smaller rotor thickness is associated with a higher critical speed. In addition, the critical speed depends on not only the master cylinder pressure but also the lining arc length. The critical speed decreased when the lining arc length and pressure magnitude increased. According to the contact pressure distribution FE results, the effective lining arc length depends on the pressure boundary condition. Therefore, for more accurate simulation, the effective lining arc length should be applied to the TEI simulation in the real brake system. The simulation results in Fig. 9 represent overestimated critical speeds except for the 2t–2t rotor. In the specimen with 80 per cent lining arc length in a two-pot caliper, two critical speeds were achieved owing to the number of hot spots. The number of hot spots depends on the lining length. From TEI theory, the critical speed is high when the number of hot spots is increased owing to an unstable thermal mode. For the occurrence of thermal instability, which can lead to the generation of a higher number of hot spots, instantaneously higher thermal energy is needed. It is impossible to have fewer hot spots than the ratio

<table>
<thead>
<tr>
<th>Component</th>
<th>Material</th>
<th>Young’s modulus (GPa)</th>
<th>Poisson’s ratio</th>
<th>Density (kg/m³)</th>
<th>Conductivity (W/m K)</th>
<th>Specific heat (J/kg K)</th>
<th>Expansion coefficient (× 10⁻⁵°C⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brake disc</td>
<td>Cast iron</td>
<td>112</td>
<td>0.25</td>
<td>7200</td>
<td>57.0</td>
<td>460</td>
<td>1.3</td>
</tr>
<tr>
<td>Brake pad, back plate</td>
<td>Steel</td>
<td>200</td>
<td>0.3</td>
<td>7850</td>
<td>42.0</td>
<td>450</td>
<td>1.2</td>
</tr>
<tr>
<td>Brake pad, lining</td>
<td>NAO (orthotropic material)</td>
<td></td>
<td></td>
<td>2377</td>
<td>2.33</td>
<td>1118</td>
<td>2.0</td>
</tr>
</tbody>
</table>
of the pad arc length to the disc circumferential length according to TEI theory [5]. Therefore, simulation results on the critical speed for various master cylinder pressures at a lining of 80 per cent area show different values as well as different tendencies from those of the experimental results. These results indicate that the critical speed is highly dependent on the pressure distribution and magni-

![Temperature distribution for various thicknesses of the brake disc, pad specimens, and pressures of the master cylinder.](image1)

**(a)** Temperature distribution on base, 2t−0t and 2t−2t rotor (2.0 bar)

**(b)** Temperature distribution on 2-pot pressurization type (left and centre: 100 per cent lining at 1.5 bar and 2 bar; right: 80 per cent lining at 2 bar)

**Fig. 8** Temperature distribution for various thicknesses of the brake disc, pad specimens, and pressures of the master cylinder. The temperature distributions show the number of hot spots. Six hot spots occurred in all cases except for the 80 per cent lining arc length, in which case there were eight hot spots for the two-pot pressurization type at a master cylinder pressure of 2 bar.

![Simulation and experimental results on the critical speed for various thicknesses of the brake disc, pad specimens, and pressures of the master cylinder respectively.](image2)

**(a)**

**(b)**

**Fig. 9** Simulation and experimental results on the critical speed for various thicknesses of the brake disc, pad specimens, and pressures of the master cylinder respectively. Full squares denote the simulation results. Crosses and asterisks show the experimental results at master cylinder pressures of 1.5 bar and 2.0 bar, respectively.
tude between the brake disc and pad. In addition, a critical pressure also exists owing to the pressure dependence. However, to consider the pressure variation between the brake disc and pads, the stiffness of the pad for different pressure magnitudes should be calculated and applied in TEI theory. In this simulation, the critical speed could not be calculated for various pressure magnitudes but for the free–free condition.

The effective lining length and pressure of the one-pot pressurization type are shorter and lower respectively than in the case of the two-pot type. In the case of a lining arc length of 39.2° with a master cylinder pressure of 2.0 bar, a rotation speed of 1000 r/min, and a two-pot pressurization type, eight hot spots are generated. In Table 5, the results on the number of hot spots and critical speeds for various pressurization types show the effect of the effective lining arc length. The effective lining arc lengths were estimated by using the relative difference of the pressure distribution through the contact pressure FE analysis results. Simulation results achieved by using the effective lining arc length can be more accurate than experimental results. In addition, it is possible to determine the caliper pressurization type reasonably. Hence, for calculating the critical speed more accurately, both the TEI and the pressure distribution and its magnitude between the brake disc and pad should be considered through correlation of the results of experiments and simulations.

4 DISC BRAKE SQUEAL

4.1 Modal analysis

It is necessary to find the natural modes and frequencies of each part of the disc brake system for analysing the natural modes and frequency of the assembled product. In particular, experiments and FE analysis on the mode shapes and natural frequencies of each part are needed in order to evaluate pad-induced squeal which is caused by coupled modes between disc and pad. Because in-plane mode frequencies of the rotor, which highly depend on the shape and size, generally occurred over 7–8 kHz, mechanical instability in the system was analysed by using correlated FE analysis results which were correlated by experimental results within 6 kHz. Therefore, in the present study, modal testing and simulations were performed to achieve out-of-plane mode shapes and frequencies of the disc and pad. A total of 150 and 80 nodal points were marked on the surfaces of brake disc and pad respectively (Fig. 10). The brake disc was excited by striking an impact hammer against it at the nodal points with a free–free boundary condition. Experimental equipment consisted of a PCB Piezotronics ICP type impact hammer and a Bruel and Kjaer 4943 type accelerometer. LMS Test.Lab was used as the modal analysis equipment. The LMS Test.Lab setting of the frequency band and range was adjusted to 1 Hz and 1–6400 Hz respectively. To perform the FE analysis, an isoparametric hexagonal mesh of the rotor and pad was used. The pad material consists of steel (isotropic) and NAO (orthotropic). The rotor material is cast iron (isotropic). The boundary conditions were the same as in the modal testing, i.e. free–free. The FE model was modified and correlated to the experimental results. Complex eigenvalue analysis was performed using the corrected FE model.

Modal tests and simulations were performed to determine the out-of-plane vibration modes of the brake disc and pad. Figures 11(a) and (b) show the out-of-plane mode shapes of the brake disc and pad respectively, as determined by impact hammer tests and FE simulations. The disc modes are characterized by the nodal circumferences and nodal diameter; the \((n, m)\) mode of the disc is characterized by \(n\) nodal circumferences and \(m\) nodal diameters. The disc is characterized by an axial symmetry; therefore the modes of the disc are generally double modes \([12]\). Tables 6 and 7 show experimental and FEA results for the out-of-plane mode for various modes \((n, m)\) including the disc thickness. The experimental and simulation results were in good agreement with those in Tables 4 and 5. Therefore, the higher modes and frequencies achieved by an FEA in comparison with those of the experimental results are also reasonable.

Table 5 Effect of effective lining arc length on the critical speed for the one-pot and two-pot pressurization type

<table>
<thead>
<tr>
<th>Caliper type</th>
<th>Pad arc length (deg)</th>
<th>Effective pad arc length (deg)</th>
<th>Pressure (bar)</th>
<th>Number of hot spots</th>
<th>Average critical speed (r/min)</th>
<th>Critical speed (TEI) (r/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>One-pot</td>
<td>46.4</td>
<td>37.1 (80%)</td>
<td>1.5</td>
<td>6</td>
<td>1300</td>
<td>835</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two-pot</td>
<td>49.8</td>
<td>44.8 (90%)</td>
<td>1.5</td>
<td>6</td>
<td>1003</td>
<td>724</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>39.2</td>
<td>35.3 (90%)</td>
<td>1.5</td>
<td>6</td>
<td>1350</td>
<td>1149</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.0</td>
<td>8</td>
<td>1000</td>
<td>1642</td>
</tr>
</tbody>
</table>
The frequency response function (FRF) curve of the rotor and pad is shown in Fig. 11(c). Since the mode shapes were determined by the impact hammer test, frequencies higher than 6400 Hz could not be excited because of the limitations of this test. Therefore, to obtain higher-order modes and to calculate complex eigenvalue problems, an FEA was performed. The results showed that, as the rotor thickness decreased, lower natural frequencies were obtained. The first bending and torsion mode frequency of the pad were 2575 Hz and 4680 Hz respectively in the modal testing results. Natural frequencies of the two-pot-type pad were lower than those of the one-pot type.

4.2 Complex eigenvalue problem

4.2.1 Theoretical formulation

The equation of motion for the brake squeal (Fig. 12) can be expressed as

$$[M] \dddot{x} + [C] \ddot{x} + [K] x = \{F\}$$

where $[M]$ is the mass matrix, which is symmetric and positive definite, $[C]$ is the damping matrix which can include friction-induced damping effects as well as the material damping contribution, $[K]$ is the unsymmetric stiffness matrix due to friction contributions between rotor and pad contact, $\{F\}$ is the force vector, and $\{\dddot{x}, \ddot{x}, x\}$ are the acceleration, velocity, and displacement vectors respectively in the system. The coupling friction force vector can be simplified as a spring connection, whose stiffness is determined by the elastic modulus of the friction material of the two pads. Therefore, the force vector can be expressed as

$$\{F\} = ([K_F] + [K_C]) \{x\}$$

where $[K_F] + [K_C]$ is the coupling stiffness matrix, referring to the relationship between the displacement vector $\{x\}$ and the coupling force vector $\{F\}$, which includes the connection force vector between brake components and the friction coupling force.

The solution of this homogeneous system is of the form

$$\{x\} = e^{\lambda t} \{\phi\}$$

where $\{\phi\}$ is a vector of complex numbers and the $\lambda$ eigenvalue is also complex in general. The eigenvalue problem for natural modes of small vibration of an FE model is

$$\lambda^2 [M] + \lambda [C] + [K_T] \{\phi\} = \{0\}$$

Fig. 10  The specimen and FE model of the brake disc and pad for modal testing. The numbers of nodal points of the disc and pad are 150 and 80 respectively.
Fig. 11 Modal testing results: (a) out-of-plane mode shape of brake disc and correlation between experiments and FE simulation; (b) out-of-plane mode shape of the brake pad (one-pot); (c) FRF curve of three rotor specimens and the brake pad.

Table 6 Experimental and FEA results of the modal analysis of the brake pad

<table>
<thead>
<tr>
<th>Component</th>
<th>Mode (out-of-plane)</th>
<th>Experiment, one-pot type</th>
<th>One-pot type</th>
<th>Two-pot type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Frequency (Hz)</td>
<td>100%</td>
<td>90%</td>
</tr>
<tr>
<td>Pad</td>
<td>First</td>
<td>2575</td>
<td>2604</td>
<td>2621</td>
</tr>
<tr>
<td></td>
<td>Second</td>
<td>4680</td>
<td>4688</td>
<td>4719</td>
</tr>
<tr>
<td></td>
<td>Third</td>
<td>–</td>
<td>6223</td>
<td>5757</td>
</tr>
<tr>
<td></td>
<td>Fourth</td>
<td>–</td>
<td>9229</td>
<td>8651</td>
</tr>
<tr>
<td></td>
<td>Fifth</td>
<td>–</td>
<td>10656</td>
<td>9708</td>
</tr>
</tbody>
</table>
where \( [M] \) is the mass matrix, which is symmetric and positive definite, \( [C] \) is the damping matrix, and \( [K_T]^5 - [K_C] \) is the stiffness matrix. By using many practical formulations such as the Lanczos method, a complex eigenvalue analysis for evaluating squeal performance can be performed by using FEA according to these equations.

### 4.2.2 FE model and conditions

To predict the braking stability, complex eigenvalue analysis provides a tool for tracing the regions of the parameter space that lead to instability of the system. In the dynamic governing equation, the negative damping force caused by friction can cause instability of the brake system. The complex eigenvalues that represented instability are extracted from eigenvector analysis. However, the limitation of the complex eigenvalue analysis is the linearized non-linearities that are only accurate near the steady state sliding state. Moreover, non-stationary features such as time-dependent material properties cannot be included. Although there are limitations, this method is effective in obtaining many potential squeal frequencies, but not all the flagged potential squeal frequencies will a problem in a real brake system [33, 34].

A real positive eigenvalue of a complex eigenvalue problem implies negative damping, which generates self-excitation or resonance in the system. Because the complex eigenvalue analysis represents an entire unstable coupled mode in the real disc brake system, the simulation results can be different from the experiments [14, 20]. However, broad unstable coupled modes and the high magnitude of negative damping indicate that a real brake system generates annoying squeal noise more easily [20]. Because the purpose of this paper suggests a main performance non-linearities that are only accurate near the steady state sliding state. Moreover, non-stationary features such as time-dependent material properties cannot be included. Although there are limitations, this method is effective in obtaining many potential squeal frequencies, but not all the flagged potential squeal frequencies will a problem in a real brake system [33, 34].

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### Table 7 Experimental and FEA results of out-of-plane mode for various disc thicknesses

<table>
<thead>
<tr>
<th>Rotor specimen</th>
<th>Mode (out-of-plane)</th>
<th>Frequency (Hz)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>(2,0)</td>
<td>1176</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>(3,0)</td>
<td>2578</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>(4,0)</td>
<td>4215</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>(5,0)</td>
<td>6007</td>
<td>1.19</td>
</tr>
<tr>
<td></td>
<td>(6,0)</td>
<td>7758</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>(7,0)</td>
<td>9604</td>
<td>–</td>
</tr>
<tr>
<td>2t–0t</td>
<td>(2,0)</td>
<td>1109</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>(3,0)</td>
<td>2351</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>(4,0)</td>
<td>3852</td>
<td>1.27</td>
</tr>
<tr>
<td></td>
<td>(5,0)</td>
<td>5517</td>
<td>2.26</td>
</tr>
<tr>
<td></td>
<td>(6,0)</td>
<td>7321</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>(7,0)</td>
<td>9156</td>
<td>–</td>
</tr>
<tr>
<td>2t–2t</td>
<td>(2,0)</td>
<td>1045</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>(3,0)</td>
<td>2171</td>
<td>2.52</td>
</tr>
<tr>
<td></td>
<td>(4,0)</td>
<td>3548</td>
<td>4.44</td>
</tr>
<tr>
<td></td>
<td>(5,0)</td>
<td>5081</td>
<td>5.71</td>
</tr>
<tr>
<td></td>
<td>(6,0)</td>
<td>6557</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>(7,0)</td>
<td>8163</td>
<td>–</td>
</tr>
</tbody>
</table>

**Fig. 12** Schematic diagram of the mathematical model for complex eigenvalue analysis of the disc brake system
Fig. 13 Simulation results of coupled mode shapes: left, rotor mode shape; centre, pad mode shape; right, coupled mode shape
evaluation and analysis for achieving optimal design factors according to the effect of disc and pad shapes, an experimental approach to the squeal phenomenon was not undertaken. Performance evaluation of the rotor and pad specimen and caliper pressurization type in the squeal phenomenon was considered about the potential unstable coupled modes through a complex eigenvalue analysis. To solve complex eigenvalue problems for each specimen, the complex eigenvalue solver of ABAQUS 6.6 was used. The following four assumptions are made.

1. The material property of each part is within the linear elastic region.
2. The friction coefficient is constant in this simulation.
3. The rotation speed of the rotor is slower than the travelling vibration wave.
4. The contact between the brake pad and rotor is in the full contact condition.

The FE model was the same as that used for the thermal and structural analysis. The friction coefficients of each specimen were 0.05, 0.1, 0.2, 0.3, 0.4, and 0.5 respectively. FE models for evaluating squeal performance were base, 2t–0t, and 2t–2t of rotor specimens; 100 per cent, 90 per cent, and 80 per cent lining arc length of one-pot pressurization type; and 100 per cent lining arc length of two-pot pressurization type. Steady state braking conditions were used; the rotation speed was 10 r/min and the contact pressure 1.5 MPa. The analysis procedure is as follows:

(a) a non-linear static analysis for brake pressurization (in this simulation, the pressure is 1.5 MPa);
(b) a non-linear static analysis to impose a rotational speed on the disc (in this simulation, \( \omega \) is 10 r/min);
(c) a normal mode analysis to extract the natural frequency of the undamped symmetric system;
(d) a complex eigenvalue analysis to find the effect of friction coupling.

![Fig. 14 Stability analysis results with various pad specimens and pressurization types for the base rotor](image-url)
Based on this procedure, complex eigenvalue problems were solved by FEA for various friction coefficients, rotor specimens, and pad specimens. Coupled mode shapes and frequencies were determined by analysing the FEA solutions.

4.3 Results and discussion

The pad-induced (or axially induced) squeal phenomenon was analysed for optimal design of the rotor and pad. It is important to note that it is only when the natural frequency of the pad is close enough to that of the disc that squeal happens [12, 35]. Strong mode coupling occurs when the normal force mode shape of one mode corresponds to the relative tangential displacement of another mode between the pad and rotor [36]. Massi et al. [35] verified that the tangential stiffness of the pad mode decreases and its natural frequency shifts to a lower frequency, becoming close to the in-plane mode of the rotor. These results indicated that the mode shapes and frequencies of the rotor are coupled by the mode shapes of the pad under the frictional contact when mechanical instability occurred. From the results of this analysis, five mode frequencies of each pad specimen under 10 kHz were found. These pad mode shapes and frequencies shifted to a lower frequency and became close to the modes of the rotor, owing to the frictional force.

Figure 13 shows the rotor and pad mode shapes, which originated from the coupled mode, and the coupled mode shapes which mainly occurred in the entire specimens. The mode shapes of the rotor coupled with each pad mode depend on the adjacency between the natural frequencies of rotor and pad. Out-of-plane mode frequencies change with the rotor thickness and this makes the adjacent frequencies change. Moreover, because the pressurization type alters the stiffness of the system, unstable modes and frequencies were slightly different from each other. Figures 14 to 16 represent stability charts which indicate the unstable char-
acteristics of the system. The frequency bands of unstable modes were similar for all cases. The coupled modes over 8 kHz, which were coupled by the in-plane mode of rotor and the fourth mode of the pad, occurred in most cases, and these are hardly affected by the disc thicknesses owing to the direction of the stiffness change. Therefore, a design change of the rotor section shape, especially the neck part, can stabilize unstable modes in this disc brake. The results for the lining arc length show that a shorter lining length, which implies a smaller contact area, decreases the unstable coupled modes. Bergman et al. [37] investigated the effects of the pad contact surface geometry and surface unit pressure on the generation of brake squeal. They concluded that the pad contact surface geometry strongly affected the occurrence and average noise level of the brake squeal. When a pressure is applied between the rotor and the pad, the contact area also becomes a constraint of the system, which changes the system’s stiffness. In addition, unstable modes can be reduced by avoiding adjacent frequencies between natural frequencies of the rotor and pad. Therefore, unstable regions, which have a positive real value, and friction coefficients, which result in coupling of the modes of the brake disc and pad, change with the brake pad stiffness and pressurization type.

The system stability of the two-pot-type caliper was relatively higher than that of the one-pot type owing to a relatively uniform pressure distribution as well as shifts in the natural frequencies. A uniform pressure distribution can reduce the mode coupling due to friction variation because, when the frictional force is reduced, the coupling effects of natural frequencies of the system will decrease according to the motion given by equation (3). Table 8 shows the unstable mode shapes, and resultant modes of the rotor specimens and 100 per cent pad of the one-pot-type caliper.

To achieve more accurate results in this system, the squeal performance evaluation method using a
complex eigenvalue problem should be correlated and compared with results obtained from experiments. According to references [14] and [21], squeal noise does not always occur in all unstable regions. However, when positive real values of the eigenvalue solution have a relatively high amplitude and are widely distributed, squeal noise can be more easily generated [34]. Therefore, changes to disc stiffness, pad shape, lining arc length, and pressurization type can improve squeal performance.

5 CONCLUSIONS

This study consisted of an experimental and FEA investigation into hot spots and squeal noise phenomena, which are responsible for the majority of brake system NVH problems. Through TEI analysis, the coning angle, the pressure distribution of lining, and a complex eigenvalue analysis for various disc thicknesses, lining arc lengths, and pressurization types were investigated to find the optimum design factors for the brake system, with particular focus on the rotor and pad. The pressure distribution between the brake disc and pad was calculated for one-pot and two-pot pressurization caliper types. It was found that, even if the pad shape and type of pressurization in the braking system are the same, the thermal and structural behaviours differ significantly owing to changes in the shape and stiffness of the rotor. In the thermal structural analysis, the 2t–0t rotor showed the smallest coning angle among all specimens. In the TEI analysis, the 2t–2t rotor displayed the highest critical speed. This implies that the rotor thickness that is relatively thinner than the pad thickness can deliver a better thermal performance. The one-pot pressurization type provided a relatively higher performance in terms of TEI than the two-pot pressurization type did owing to a more uniform pressure distribution. Therefore, changes to disc stiffness, pad shape, lining arc length, and pressurization type can improve squeal performance.

However, on consideration of the effective lining arc length and pressure, the TEI performance showed nearly the same tendency for one-pot and two-pot pressurization. In addition, two-pot pressurization could provide a higher performance with respect to braking stability than the one-pot type can, because the uniform pressure distribution of the former resulted in more stable braking.

The analysis results of the squeal phenomenon through a complex eigenvalue problem for various rotor thicknesses showed that a reduced pad has a less unstable region and a relatively high friction coefficient. When the lining arc length was reduced, the braking system stability was slightly changed. In particular, 80 per cent lining arc length provided the best performance of the one-pot pressurization-type pad specimens. System stability of the two-pot-type caliper was relatively higher than that of the one-pot type owing to the uniform pressure distribution as well as shifts in the natural frequencies of the pad. This implies that the uniform pressure distribution makes the system more stable, e.g. being able to predict the result by the governing equation of motion. In the results for various rotor thicknesses, unstable modes dominantly occurred owing to the differences between the adjacent mode frequencies of the rotor and pad due to the change in the out-of-plane mode frequencies. Therefore, squeal performance is affected not only by the system but also by the uniform pressure distribution, boundary conditions, and stiffnesses of the rotor and pad.

Optimal design through consideration of the coupled thermal and mechanical analysis is highly important to the brake NVH as well as to the fundamental performance of the brake system. Because the optimal design factors are coupled to each other, coupling analysis at the initial step of disc brake design should be considered and performed to develop a disc brake system of lower cost and higher performance.

Table 8 Complex eigenvalue analysis results according to the rotor specimens at 100 per cent pad of one-pot type caliper

<table>
<thead>
<tr>
<th>Rotor specimen</th>
<th>Coupled mode (rotor mode + pad mode)</th>
<th>Coupled mode frequency (Hz)</th>
<th>Real part</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>(2.0) + first</td>
<td>1284</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>(6.0) + third</td>
<td>7575</td>
<td>274.3</td>
</tr>
<tr>
<td></td>
<td>8613 Hz + fourth</td>
<td>8509</td>
<td>311</td>
</tr>
<tr>
<td>2t–0t</td>
<td>4474 Hz + second</td>
<td>5041</td>
<td>399</td>
</tr>
<tr>
<td></td>
<td>(6.0) + third</td>
<td>7215</td>
<td>132.4</td>
</tr>
<tr>
<td></td>
<td>7321 Hz + fourth</td>
<td>7735</td>
<td>386</td>
</tr>
<tr>
<td></td>
<td>8475 Hz + fifth</td>
<td>8613</td>
<td>446</td>
</tr>
<tr>
<td>2t–2t</td>
<td>(2.0) + first</td>
<td>1118</td>
<td>19.6</td>
</tr>
<tr>
<td></td>
<td>8400 Hz + fifth</td>
<td>8463</td>
<td>71.1</td>
</tr>
<tr>
<td></td>
<td>8543 Hz</td>
<td>8785</td>
<td>569</td>
</tr>
</tbody>
</table>
ACKNOWLEDGEMENT

This work was supported by Inha University.

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